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Additional Remarks on the Principle of Equivalence, Electrodynamics and General Relativity

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This is a brief note meant to clarify and strengthen the premise of a paper written by this author and entitled, 'The Principle of Equivalence, Electrodynamics and General Relativity', which was published in an earlier issue of this journal (Cohn, 1969).

The purpose of the above work was to prove that the principle of equivalence and the covariant form of Maxwell's equations lead to markedly different predictions for the electric field immediately outside a straight resistanceless current-carrying wire at rest in a weak gravitational field produced by a mass M. It was shown that the principle of equivalence leads to a field depending on $\alpha = 2GM/r_0 C^2$ to first order, whereas the covariant Maxwell's equations lead to an electric field which is zero to first order in α.

In demonstrating that E_1 (component of the electric field along the wire) was zero to first order in α, we neglected all effects coming from the purely electromagnetic contributions to the metric. This procedure is valid providing that the contributions to E_1 (to all orders) arising from just the mass M are not all zero. This contingency, however, was not treated in the cited paper, and we shall briefly consider it here.

The covariant Maxwell equations relating the vector potential A_μ and current-density 4-vector j_μ is given by equation (3.12) in the previous paper as

$$g^{\alpha\beta} A_{\mu;\beta\alpha} + g^{\alpha\beta} B^{\epsilon}_{\beta\alpha\mu} A_\epsilon = \frac{4\pi}{C} j_\mu \tag{1}$$

If we neglect all contributions to the metric coming from the electromagnetic field surrounding the wire, this equation becomes

$$g^{\alpha\beta} A_{\mu;\beta\alpha} = \frac{4\pi}{C} j_\mu \tag{2}$$

where the involved metric only depends now on M.

Setting $\mu = 4$, we then have

$$g^{\alpha\beta} A_{4;\beta\alpha} = \frac{4\pi}{C} j_4 \tag{3}$$

Now the physical situation we have is such that, if $E_1 = -\partial A_4/\partial x_1$ is to be non-zero at a point just outside the midpoint of the wire, then our equation must depend on a direction dependent source quantity, like j_i ($i \neq 4$). However, the above equation does not; and as we can see without further detail, by symmetry we must have $E_1 = -\partial A_4/\partial x_1 = 0$, at a point just outside the wire's midpoint. This statement applies, of course, without reference to any particular order of α.

That is, the contribution to E_1 arising from just the mass M, is zero at the point of interest.

Now we are forced to include the contribution to the metric coming from the electromagnetic field about the wire.

Referring to equation (1), and using the Field Equations

$$B_{\mu\nu} = -\frac{8\pi G}{C^4} T_{\mu\nu} \equiv -\kappa T_{\mu\nu} \tag{4}$$

(instead of $B_{\mu\nu} = 0$ as before) we obtain

$$g^{\alpha\beta} A_{\mu;\beta\alpha} - \kappa A^{\lambda} T_{\mu\lambda} = \frac{4\pi}{C} j_{\mu} \tag{5}$$

Now setting $\mu = 4$ then gives

$$g^{\alpha\beta} A_{4;\beta\alpha} - \kappa A^{\lambda} T_{4\lambda} = \frac{4\pi}{C} j_4 \tag{6}$$

This equation might give a non-zero $\partial A_4/\partial x_1$ at the point of interest, since A_4 is coupled to the other A^{λ}, i.e. to the other components j^{λ}. Concerning orders of magnitude, note that we may neglect electromagnetic contributions to the metric in the first term of this equation since they are comparatively very small. Furthermore, from the expansion

$$A_4 = A_4^{(0)} + \alpha A_4^{(1)} + \cdots \tag{7}$$

we can write

$$T_{4\lambda} = T_{4\lambda}^{(0)} + \alpha^2 T_{4\lambda}^{(1)} + \cdots \tag{8}$$

where quantities with a zero superscript refer to the quantities when all gravitational effects are absent. Now, $T_{4\lambda}^{(0)} = 0$ (for $\lambda \neq 4$) since $\mathbf{E} = 0$ outside the superconductor, in the classical case. Therefore, through first order in α we have

$$g^{\alpha\beta} A_{4;\beta\alpha} - \kappa A^4 T_{44}^{(0)} = \frac{4\pi}{C} j_4 \tag{9}$$

where the metric depends only on terms through first order in α.

But in this equation, as in equation (3), there is no dependence on directionally dependent quantities, so that once again $E_1 = -\partial A_4/\partial x_1 = 0$ at the point of interest; thus, through first order in α there is no contribution to E_1 (at the point of interest) arising from the *total* metric.

Therefore, we arrive at the same conclusion as that drawn in the preceding paper, but from a more rigorous point of view.

References

Cohn, J. (1969). *International Journal of Theoretical Physics*, Vol. 2, No. 2, p. 125.